Proc. Nat. Acad. Sci. USA
Vol. 71, No. 12, pp. 4884–4886, December 1974

Reconstruction of Objects from Radiographs and the Location of Brain Tumors

(transaxial tomography)

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Communicated by R. H. Bing, April 25, 1974

ABSTRACT **The objective of this research is two-fold: (*i*) the development of a method for locating hemorrhages, blood clots, and tumors in the brain by means of ordinary hospital radiographs and without the introduction of contrast material; and (*ii*) the uncovering of new mathematical results through this point of view. The present note deals mainly with the first objective, but the mathematical setting and convergence and indeterminacy theorems are also described.**

Remarkable advances have been made recently in localizing brain masses and other pathologies from radiographs without the introduction of contrast material. In particular, there are the EMI and Georgetown scanners, and new ones continue to be devised. The aim of our work is rather different, namely to do what can be done with ordinary equipment that is readily available to every hospital. The equipment we use consists of a swivel chair with an accurate ratchet, a head holder, and an aluminum wedge. Eighteen radiographs are taken at 10° angles around a semi-circle. They are read by a densitometer and analyzed on a computer by a method similar to that of the EMI scanner, and horizontal cross sections are produced at any desired level. In this note are presented reconstructions of three cross sections of a pig's head (Fig. 1) and three cross sections of the head of a patient with a brain tumor (Fig. 3).

MATHEMATICAL SETTING

The mathematical setting of the reconstruction problem is as follows. An object in R^n is determined by a density function f, $f(x)$ being the density at the point x. A radiograph from a direction θ provides a function $P_\theta f$ on the plane orthogonal to θ whose value at a point z of this plane is the total mass along the line through z in the direction θ:

$$P_\theta f(z) = \int_{-\infty}^{\infty} f(z + t\theta)dt. \qquad [1]$$

The problem is the recovery of the unknown density f from the knowledge of a certain number of the radiographs $P_{\theta_1}f, \ldots, P_{\theta_M}f$. It is immediately seen that a solution to the 2 dimensional problem gives a solution to the general problem, for it recovers all 2 dimensional sections orthogonal to a given $n - 2$ dimensional axis. In 2 dimensions, the x-ray transform is effectively the same as the Radon transform, for which explicit inversion formulas are available, though not in a form to be very useful in practice. It will be assumed throughout that the density functions are square integrable with compact support.

Reconstruction methods can easily be devised. The one we have used is a simple iterative scheme that was thought at first to be new but turned out to be an adaptation of the classical method of Kacmarz (1) for the solution of a system of linear equations. The Kacmarz method is as follows. Suppose that N_j is the null space of P_{θ_j}, that f is the true solution, that P_j is the orthogonal projection (in L^2) on $f + N_j$, and that $P = P_M \ldots P_1$. The method consists in choosing an initial guess f_0 and setting $f_n = P^n f_0$. The P_j are computable, and it is shown in (2) that if $N = \bigcap N_j$, then the f_n converge in the L^2 sense to the projection of f_0 on $f + N$. This Kacmarz method is effectively the one used also by the EMI scanner. We have established the following rate of convergence.

If each N_j makes an angle $\geq \alpha$ with the intersection of the following ones and f' is the projection of f_0 on $f + N$, then

$$||f_n - f'||^2 \leq (1 - \sin^{2M-2}\alpha)||f - f_0||^2.$$

It seems plausible that the angle α is approximately equal to the minimum angle between the x-ray directions, but this is by no means proved.

The objects in the plane $f + N$ have precisely the same radiographs from the given directions, and are therefore indistinguishable by radiographs from these directions. The question of the actual possible distinctions between these objects has been largely overlooked. We have the following theorem to show that the radiographs by themselves give no information whatever about the interior of the object.

Suppose there are given an infinitely differentiable object f_0 and a finite number of directions. Then there is a new object f with exactly the same shape, exactly the same radiographs from these directions, and a completely arbitrary infinitely differentiable density distribution on any compact set in the interior of the support of f_0.

The practical consequence of this theorem (and it is supported by experimental evidence) is that each kind of reconstruction problem must be approached on its own and afresh. Many problems appear to be accessible by the same basic idea (e.g., other parts of the body, cell structure, defects in nuclear fuel assemblies, etc.) but each must be examined carefully from the point of view of picking out the right object in the plane $f + N$.

A number of purely mathematical theorems have resulted from this x-ray point of view, but they will be presented elsewhere.

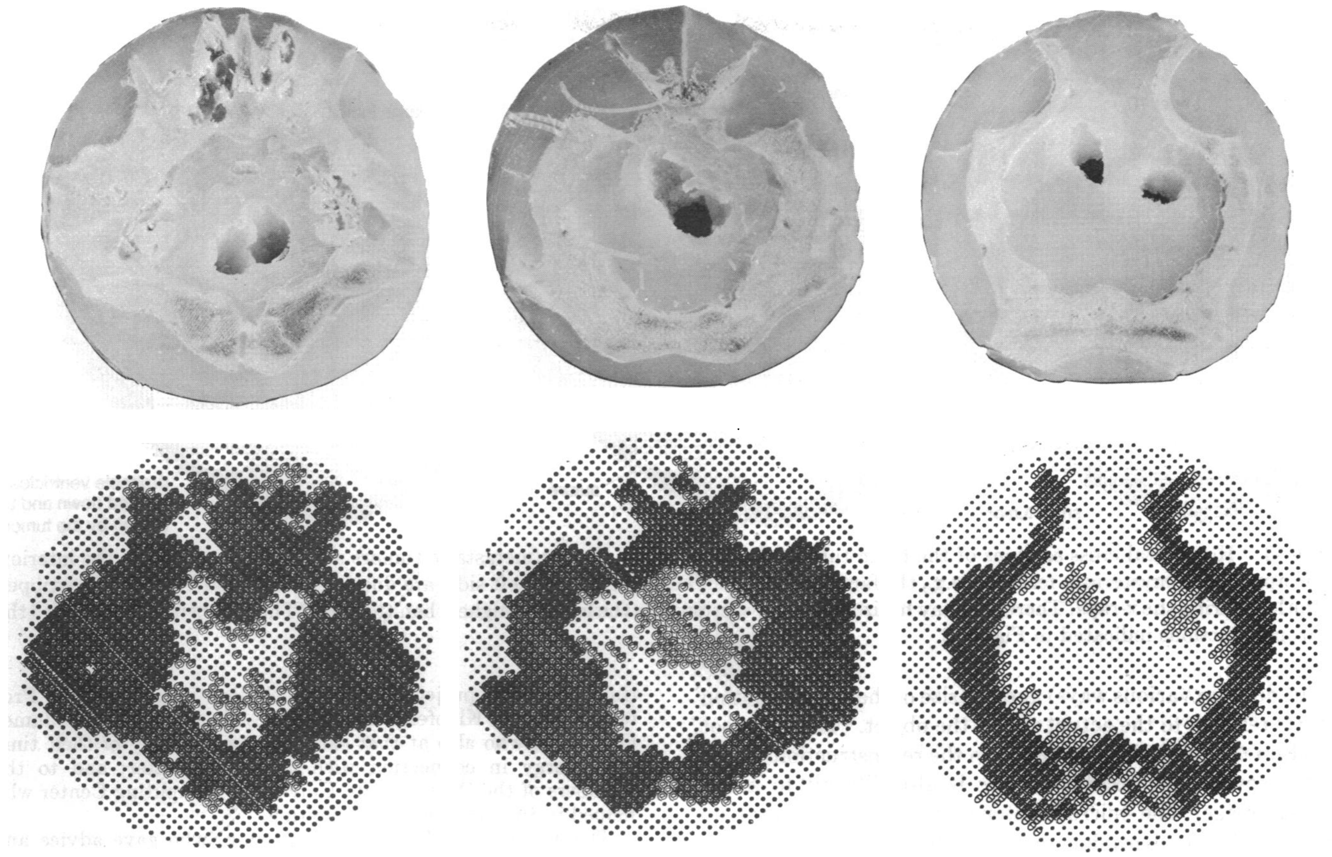

FIG. 1. Reconstruction of pig's head. The pig's head was a phantom consisting of a real skull about 10 cm in diameter filled with tissue equivalent wax (49% beeswax, 49% paraffin, and 2% resin) in which there were two water filled holes about 0.8 cm in diameter. In the *upper row* are actual photographs of cross sections of the pig's head. In the *lower row* are reconstructions of the same cross sections.

EXPERIMENTAL RESULTS

After successful preliminary tests on a tooth and on the pig's head (Fig. 1) the method was used on a brain tumor patient (Figs. 2 and 3). The reconstructions showed clearly the skull, the brain, the metastatic tumors in locations confirmed by cerebral angiography, and probably the ventricles. The radiographs were taken with ordinary hospital equipment and without the introduction of contrast material.

In the case of the brain tumor patient the technical details were as follows. The shots were taken at 85 kv and 20 to 40 mA sec with a 0.3 mm focal spot. The films used were Kodak RPR, and they were developed by the hospital automatic developer. It should be understood that these details are tentative. Analysis of the data from the brain tumor patient and subsequent experiments indicate that the energy should be raised a little.

An important feature in the experiments was a calibration wedge in each radiograph. The densitometer reading at a point z of the radiograph is not at all the number $P_\theta f(z)$ given by Eq. 1, but is related to it by a complicated formula that changes with the strength of the x-ray. However, if $w(x)$ denotes the densitometer reading at a distance x from the edge of the wedge and if w^{-1} is applied to all the data, then the resulting numbers are (proportional to) the true total masses $P_\theta f$. Thus, the wedge not only corrects for the nonlinearity in the relation between the densitometer readings and the true total masses, but also for variations (both intentional and unintentional) in the strengths of the several x-rays.

Some additional technical details and comments can be found in (3).

DISCUSSION

At the present early stage it is impossible to compare the quality of the reconstructions with that obtainable by the scanners. In the case of the brain tumor patient, the existence and location of the tumor was confirmed by angiography. In the case of the pig's head, the reconstructions can be compared to the photographs of the actual cross sections, but many improvements have been made since the pig's head was done. The major factors affecting resolution appear to be the indeterminacy described in the theorem above (which can

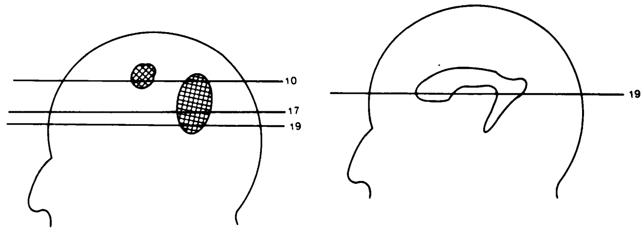

FIG. 2. Location of brain tumors (as demonstrated by angiography) in a patient. The profile sketches of the patient indicate the levels of the cross sections shown in Fig. 3. On the left profile are indicated a small tumor on the left and a large tumor on the right. On the right profile are indicated the ventricles. The numbers correspond to the cross sections shown in Figure 3.

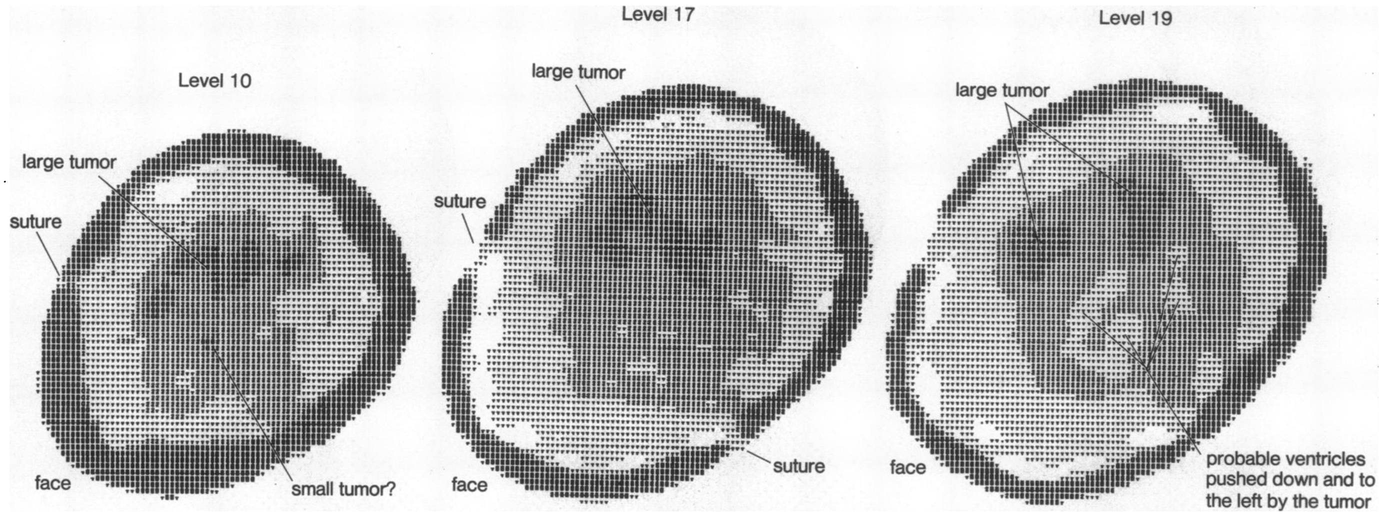

FIG. 3. Horizontal cross sections of the brain tumors sketched in Fig. 2. The metastatic tumors are the dark masses in the interior. Some apparently mysterious features, e.g., the fact that the skull is thicker on the left side, are due to the fact that the head was tipped to the left during the x-rays. Therefore, any horizontal cross section cuts the left side higher than the right. The numbers correspond to the horizontal sections indicated in Figure 2.

be cut down in various ways) and the size of the reconstruction matrix relative to the actual size of the object. (The tiny root canals in the tooth showed clearly in the reconstructions.)

The cost of the reconstructions is also difficult to assess at this stage. The microdensitometer used necessitates an unfortunate photographic reduction to 5×5 photographic film, requires an excessive amount of operator time, and creates location and matching problems that are extraneous. Exclusive of such costs, the computer time has been about three minutes on the Control Data Corporation 3300 computing machine. Programs now being tested will probably reduce this to one or two minutes.

There is a substantial literature on the reconstruction problem. Attention is directed to (4) where the Kacmarz method is suggested for use with x-rays and to subsequent papers by the same authors that trace the history of the problem in various fields. An extensive bibliography has been prepared by Dr. Richard Gordon of the National Institutes of Health.

This research has been supported by the National Science Foundation Grant GJ 37325.

Particular thanks are due to Dr. Cyrus W. Austin, who took the initial x-rays of the tooth (on his dental x-ray machine) at a time when our project was very nebulous indeed; and to Professors Charles Klopfenstein and James Koskinen and Mr. James Crossland, who also at that early date gave freely of their time and energy in connection with the densitometer; and to the members of the Oregon State University Computing Center who helped in too many ways to mention.

Thanks are also due to the following, who gave advice and assistance in their own particular fields: Mr. John Kelly of the O.S.U. x-ray department; Mr. Pete Meyer and Professor Larry Schechter of the O.S.U. physics department; Miss Sally Browning of the University of Oregon Computing Center; Mr. Harry Howard of the U.O. biology department photography lab; Mr. Thomas Hutchins, free lance inventor of Portland; and Dr. William Bank of the U.O. Medical School in Portland.

Many of the ideas in this work arose in discussions with colleagues in the O.S.U. mathematics department.

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